Name Keanu Vegt

CALCULUS BC

SECTION II - A

EXAM - CHAP. 3 + 4

18/27

SHOW ALL YOUR WORK. INDICATE CLEARLY THE METHODS YOU USE BECAUSE YOU WILL BE GRADED ON THE CORRECTNESS OF YOUR METHODS AS WELL AS ON THE ACCURACY OF YOUR FINAL ANSWERS.

Notes: (1) In this examination $\ln x$ denotes the natural logarithm of x (that is, logarithm to the base e). (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. A particle starts at time $t = 0$ and moves along the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = (t-1)^3(2t-3)$.

   (a) Find the velocity of the particle at any time $t \geq 0$.

   (b) For what values of t is the velocity of the particle less than zero? Justify your answer

   (c) Find the value of t when the particle is moving and the acceleration is zero.

$x(t) = (t-1)^3(2t-3),\ t \geq 0$

a) $v(t) = x'(t)$

$\boxed{v(t) = 3(t-1)^2(2t-3) + 2(t-1)^3}$ ✓

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b) $V'(t) < 0$

$v(t) = 3(t^2-2t+1)(2t-3) + 2(t^3-2t^2+t-t^2+2t-1)$

$= (3t^2-6t+3)(2t-3) + 2(t^3-3t^2+3t-1)$

$= 6t^3 - 12t^2 + 6t - 9t^2 + 18t - 9 + 2t^3 - 6t^2 + 6t - 2$

$= 8t^3 - 27t^2 + 30t - 11$

−3 ?

$V(t) = 0$, when $t = 1$. Thus, ~~since this is a "positive cubic function"~~ $\boxed{v(t) < 0 \text{ when } x \in (0, 1)}$

c) $v(t) \neq 0$ and $a(t) = 0$.

$a(t) = v'(t)$

$= 24t^2 - 54t + 30$

$= 6(4t^2 - 9t + 5)$

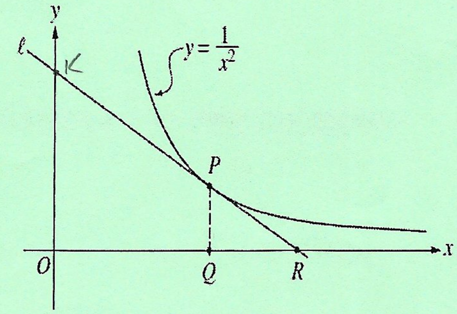
$a(t) = 0$

$6(4t^2 - 9t + 5) = 0$

$t = \frac{9 \pm \sqrt{81-80}}{8} = \frac{9 \pm 1}{8}$ $\cancel{t = 1}, \frac{5}{4}$

$t = 1$ is extraneous because it conflicts with the requirement of $v(t) \neq 0$. Therefore $\boxed{t = \frac{5}{4}}$

2. In the figure above, line ℓ is tangent to the graph of $y = \dfrac{1}{x^2}$ at point P, with coordinates $\left(w, \dfrac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at the point R, with coordinates $(k, 0)$.



(a) Find the value of k when $w = 3$.

(b) For all $w > 0$, find k in terms of w.

(c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

(d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

a) $\left(3, \frac{1}{9}\right)$ $\quad m = \frac{-2}{27}$ $\quad$ $1/9$

$\left(y + \frac{1}{9}\right) = \frac{-2}{27}(x-3)$

$y = \frac{2}{9} + \frac{1}{9}$

$y = \frac{1}{3} \Rightarrow \boxed{\cancel{K = \frac{1}{3}}}$ $\quad -1$

b) $K = \frac{-2}{w^2} + \frac{1}{w^2} = \boxed{\frac{3}{w^2}}$ $\quad -2$

c) $K' = \frac{-6}{w^3} \cdot 7 \Rightarrow \frac{-6}{125} \cdot 7 = \boxed{-\cancel{\frac{42}{125}}}$ $\quad -1$

d) no time left.

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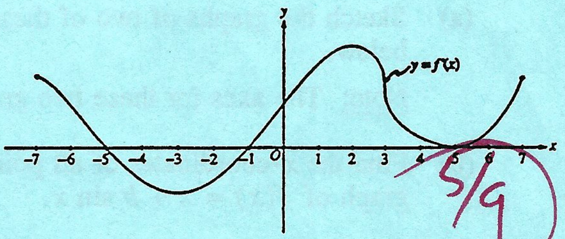
SECTION II - PART B. NAME Keann Vestil

3. The figure above shows the graph of f', the derivative of the function f, for $-7 \le x \le 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.



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(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

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f' ? which derivate?

a) $\boxed{x=-1}$, it is the only point where the derivative crosses from negative to positive -1

b) $\boxed{x=-5}$, it is the only point where the ~~derivative crosses~~ from positive to negative

c) $x \in (-7,-3), (2,3), (3,5)$, these are the only intervals where $f'(x)$ is decreasing

d) $\boxed{x=7}$, the area under the curve from $x=-1$ (rel. min) to $x=7$ is greater than that of $x=-7$ to $x=-1$

?

-2

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4. This problem deals with functions defined by $f(x) = x + b \sin x$, where b is a positive constant and $-2\pi \le x \le 2\pi$.

(a) Sketch the graphs of two of these functions, $y = x + \sin x$ and $y = x + 3 \sin x$, as indicated below.

<u>Note:</u> The axes for these two graphs are provided in the pink test booklet only.

(b) Find the x-coordinates of all points, $-2\pi \le x \le 2\pi$, where the line $y = x + b$ is tangent to the graph of $f(x) = x + b \sin x$.

(c) Are the points of tangency described in part (b) relative maximum points of f? Why?

(d) For all values of $b > 0$, show that all inflection points of the graph of f lie on the line $y = x$.

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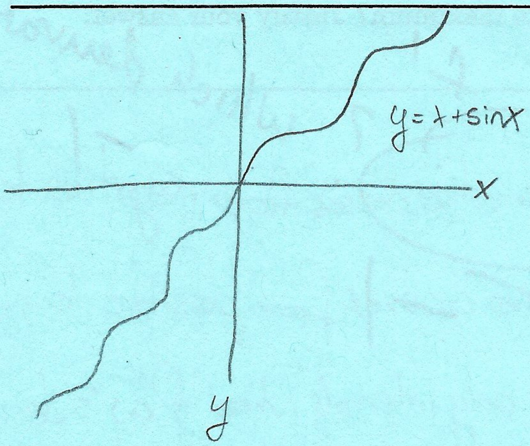
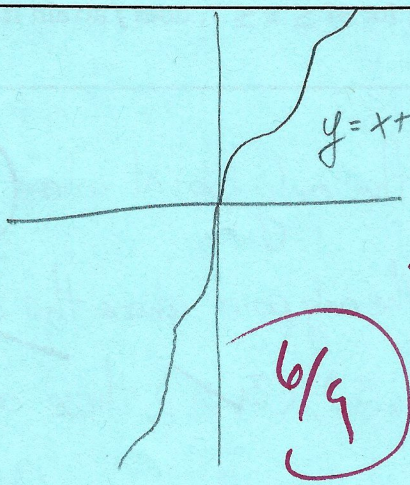
a)





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b) $(x+b)' = (x+b\sin x)'$, $-2\pi \le x \le 2\pi$

$\Downarrow$

$1 = 1 + b\cos(x)$

$b\cos(x) = 0$

$\boxed{x = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}}$

-2

c) $\boxed{\text{no}}$, because the function is still increasing after these points. It possesses no relative extrema.

d) $f'(x) = 1 + b\cos(x)$

$f''(x) = -b\sin(x)$

$f''(x) = 0 = -b\sin(x)$

$x = -2\pi, -\pi, \pi, 2\pi$

plugging them into the original f(x) yields:

$f(-2\pi) = -2\pi + b(0)$

$f(-\pi) = -\pi + b(0)$

$f(\pi) = \pi + b(0)$

$f(2\pi) = 2\pi + b(0)$

therefore, all points of inflection of f(x) lie on y=x.

nice.