Keanu Vestil
Period 2

Chapter 3 and 4 Summary

Chapter 3

★ Do not use product/quotient rule with constants

★ Try to use algebra, trig, or precalc before Calculus.

★ Since calculus cannot handle absolute values, must create piecewise-defined f(x)

Logarithmic Differentiation ?

$\frac{d}{dx}(\ln(x)) = \frac{1}{x}$

$\frac{d}{dx}(\log_b(x)) = \frac{d}{dx}\left(\frac{\ln(x)}{\ln(b)}\right) = \ln(b)\frac{d}{dx}(\ln(x))$

★ Ignore absolute values with logarithmic functions like $\ln|x|$

10

Implicit Differentiation

ex. $\frac{d}{dx}(y^2 + x = y + x^2) \Rightarrow 2y\frac{dy}{dx} + 1 = \frac{dy}{dx} + 2x$

· Define function in terms of x and y, implying the original function with "y". $y' \Rightarrow \frac{dy}{dx}$

Exponential Differentiation

· Take ln of both sides to apply logarithim identities (ex $\ln x^e = e\ln x$)

ex. $\frac{d}{dx}\left(e^y\right) = e^y \cdot \frac{dy}{dx}$

· Then solve for $\frac{dy}{dx}$ algebraically

· natural logs are your friends ? :)

Inverse Differentiation

· Given coordinate pair (a, b), domain and range interchange

$f^{-1\prime}(b) = \frac{1}{f'(a)}$

$\frac{d}{dx}\left(\sin(x)^{-1}\right) = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\left(\cos(x)^{-1}\right) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left(\tan(x)^{-1}\right) = \frac{1}{1+x^2}$, $\frac{d}{dx}\left(\cot(x)^{-1}\right) = \frac{-1}{1+x^2}$

$\frac{d}{dx}\left(\sec(x)^{-1}\right) = \frac{1}{|x|\sqrt{x^2-1}}$, $\frac{d}{dx}\left(\csc(x)^{-1}\right) = \frac{-1}{|x|\sqrt{x^2-1}}$

Related Rates

Given a rate, $\frac{dV}{dt}$, want to find $\frac{dR}{dt}$, use $\frac{dV}{dt} \cdot \boxed{\frac{dR}{dV}} = \frac{dR}{dt}$

Solve for $\frac{dR}{dt}$

Chapter 4

★ extrema are y-values
★ always use radians
★ when rounding, use three decimals

Optimization

Step 1: maximizing or minimizing?
Step 2: what is getting maximized/minimized?
Step 3: formula for Step 2.
Step 4: write formula in 1 variable using auxilary equations

2nd and 3rd derivative

$f'(x)>0$, $f(x)$ increasing
$f'(x)<0$, $f(x)$ decreasing
$f''(x)>0$, $f(x)$ concave up
$f''(x)<0$, $f(x)$ concave down

- when defining increasing/decreasing, [a, b]
- when defining concavity, (a, b)

Mean Value Theorem

"Iff $f(x)$ is continuous on [a, b] and differentiable on (a, b), then $\exists c \mid f'(c)=\frac{f(b)-f(a)}{b-a}$." (A point where instant rate of change is equal to average rate of change)

Rolle's Theorem (subset of Mean Value Theorem)

"Iff $f(x)$ is continuous on [a, b] and differentiable on (a, b) and $f(a)=f(b)$, then $\exists c \mid f'(c)=0$." (Since the average rate is 0, a point where the tangent is also horizontal)

Rectilinear Motion

$s(t)$ is position, $v(t)$ is velocity, $a(t)$ is acceleration
$s'(t)=v(t)$
$s''(t)=v'(t)=a(t)$
$\int_a^b v(t)$ is displacement, $\int_a^b |v(t)|$ is total distance travelled
Use geometric method for integrals with basic shapes