AP Calculus
Smith

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$\frac{24}{30}$

Exam-Chapter Five
Section II-Part A

A graphing calculator <u>IS NEEDED</u> on this section of the exam.

Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

$\frac{8}{9}$

a) $G'(t) = -45\sin\left(\frac{t^2}{18}\right)\cdot\frac{t}{9}$

$G'(5) = -45\sin\left(\frac{25}{18}\right)\cdot\frac{5}{9} \approx -24.588 \frac{\frac{\text{ton}}{\text{hour}}}{\text{hour}}$

Since $G(x)$ is the rate at which it arrives $G'(x)$ is the rate at which the arrival rate changes. Since $G'(5)$ is negative, this means that there is a ~~decreasing amount of gravel arriving~~ at hour 5. -1

"-24.588 tons per hour squared"

b) $\int_0^8 G(t)\,dt = \boxed{825.551 \text{ tons of gravel}}$

c) $G(t) - 100 \rightarrow$ rate of arrival − rate of process = rate of net amount

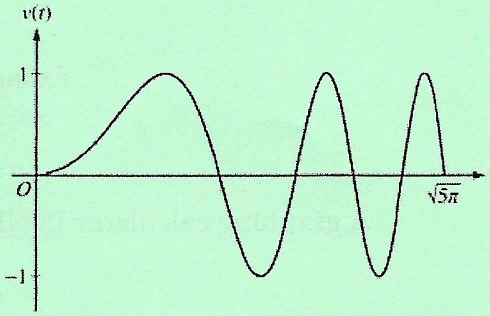
$\boxed{(G(5)-100) < 0, \text{ the amount of unproceffed grave is decreasing at } t=5}$

d) $G(t) - 100 = 0$ at $t_{max} = 4.923$ crossing from positive to negative, therefore it is a relative maximum

$\int_0^{t_{max}} (G(t)-100)\,dt + 500 = \boxed{635.376 \text{ tons of gravel is the max}}$

Question 2

A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2)$. The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.



(a) Find the acceleration of the particle at time $t = 3$.

(b) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.

(c) Find the position of the particle at time $t = 3$.

(d) For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to the right. Explain your answer.

a) $v'(3) = a(3)$

$v'(3) = \boxed{-5.467}$

b) $\int_0^3 |v(t)|\,dt = \boxed{1.702}$

c) $5 + \int_0^3 v(t)\,dt = \boxed{5.774}$

d) no time again ☺

4/9

, 3

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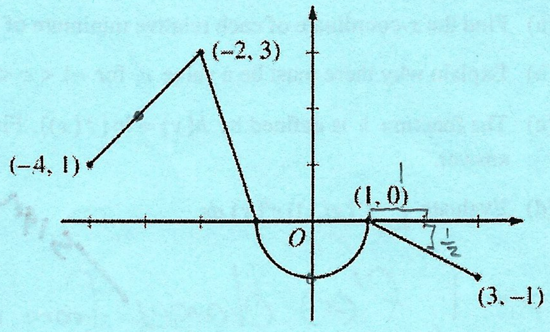
Exam-Chapter Five
Section II-Part B

A graphing calculator is NOT ALLOWED on this section of the exam.

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

5/9

a) $g(2) = \boxed{-\frac{1}{4}}$

$g(-2) = \frac{\pi}{2} - \frac{3}{2} = \boxed{\frac{\pi - 3}{2}}$

b) $g'(-3) = \boxed{2}$

$g''(-3) = \boxed{1}$

Why? −1

c) g(x) has a horizontal tangent at $\boxed{x=-1 \text{ and } x=1}$, because $g'(x)=f(x)$, $f(x)=0$ at -1 and 1

x=−1 is a relative maximum, x=1 is not a maxima or minima. ?

d) g(x) has a point of inflection at x=0, because $g'(x)=f(x)$, $g''(x)=f'(x)$, $f'(x)=0$ at $x=0$

−3

Question 4

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1	$1 < x < 3$	3
$f(x)$	12	Positive	8	Positive	2	Positive	7
$f'(x)$	-5	Negative	0	Negative	0	Positive	$\frac{1}{2}$
$g(x)$	-1	Negative	0	Positive	3	Positive	1
$g'(x)$	2	Positive	$\frac{3}{2}$	Positive	0	Negative	-2

The twice-differentiable functions f and g are defined for all real numbers x. Values of f, f', g, and g' for various values of x are given in the table above.

(a) Find the x-coordinate of each relative minimum of f on the interval $[-2, 3]$. Justify your answers.

(b) Explain why there must be a value c, for $-1 < c < 1$, such that $f''(c) = 0$.

(c) The function h is defined by $h(x) = \ln(f(x))$. Find $h'(3)$. Show the computations that lead to your answer.

(d) Evaluate $\int_{-2}^{3} f'(g(x))g'(x)\,dx$.

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a) at $x = 1$, $f'(x)$ changes from negative to positive, therefor a relative minimum exists here.

how do you know?

b) According to Rolles' Theorem, iff a function is continuous and differentiable on $[a, b]$ and the average rate of change is 0, there must exist a point at which the derivative, in this case $f''(c)$, is also 0. -2

c) $h'(x) = \frac{1}{f(x)} \cdot f'(x)$, $h'(3) = \frac{1}{7} \cdot \frac{1}{2} = \boxed{\frac{1}{14}}$

d) $2 - 8 = \boxed{-6}$ how?

-2