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Chapter 5 Summary

- Indefinite Integrals (Antiderivative)

★ Derivative is a limit
★ Integral is a limit

limits $\int_a^b \underbrace{f(x)\,dx}_{\text{Integrand}}$

Integral

Power Rule:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Natural Log case:

$$\int \frac{1}{x} dx = \ln|x| + C$$

U-substitution:

Define a part of the integrand as "u" in order to cancel out other variables.

ex: $\int (2x+1)^3 dx \quad u = 2x+1$

$$\frac{du}{dx} = 2$$

$$\int u^3 \frac{du}{2} = \frac{1}{2}\int u^3 du = \frac{1}{2}\cdot\frac{u^4}{4} = \frac{(2x+1)^4}{8} + C$$

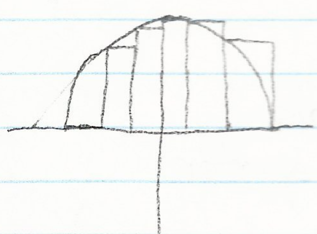
Constants can be moved:

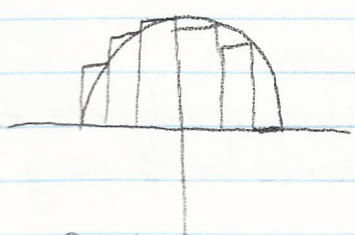
$$\int k f(x)dx = k\int f(x)dx$$

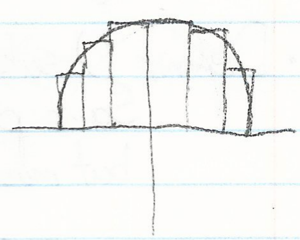
Addition/Subtraction Rule is your best friend:

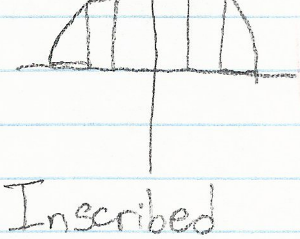
$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$

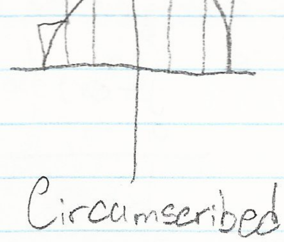
- Integral Approximation


Left-Sum


Right-Sum


Midpoint


Inscribed


Circumscribed

- Add together areas of rectangles to approximate the area under a curve.

• Definite Integrals

$$\int_a^b f'(x)dx = f(b)-f(a), \quad \int_a^b f''(x)dx = f'(b)-f'(a)$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b k f(x)dx = k\int_a^b f(x)dx$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\int_{g(x)}^{a} f(t)\,dt = -f(g(x))\cdot g'(x)$$ all cases.

• Definit U-substitution

Same procedure as indefinite integrals, but must rewrite limits using u equation

• Mean Value Theorem

iff $f(x)$ is continuous on $[a,b]$, the average value over an interval is

$$\frac{\int_a^b f(x)dx}{b-a}$$