Free Response

Directions: Please show all relevant work in a clear and concise manner. Failure to show work may result in a loss of credit. Also …

1. each question will be graded out of 9 points
2. all answers are to be exact or rounded to three decimal places
3. use interval notation where appropriate

3/

1. Solve the following rational inequality algebraically and verify your solution using a graph. Make sure that all relevant points are labeled on the graph.

$$\frac{2x}{x-2} \le 3$$

$$\frac{2x}{x-2} - 3 \le 0$$

x-int: $\frac{2x}{x-2} = 0$

$x = 0$

x-int: $\frac{2x}{x-2} - 3 = 0$

$\frac{2x}{x-2} = 3$
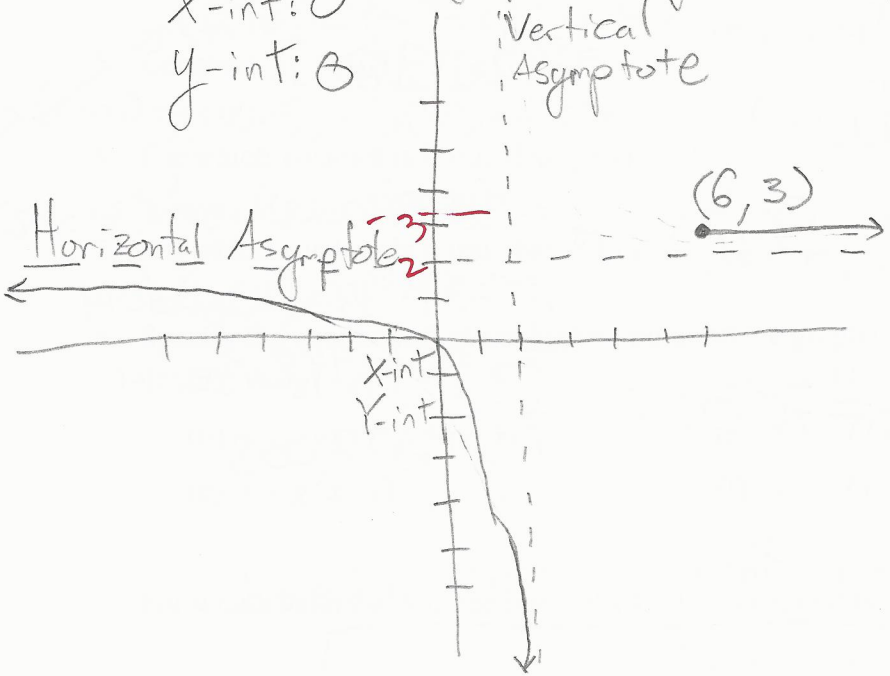
don't understand what you're doing here

$2x = 3x - 6$

$x = 6$

At $x=6$, $y=0$

Thus, in $\frac{2x}{x-2}$, At $x=6$, $y=3$

$$(-\infty, 2) \cup [6, \infty)$$

Vertical Asymptote: $x = 2$
Horizontal Asymptote: $y = 2$
X-int: 0
y-int: 0

Horizontal Asymptote $\frac{3}{2}$

(6,3)

Vertical Asymptote

X-int
Y-int

2. A Norman window is in the shape of a rectangle surmounted by a semicircle, as shown in the figure. Assume that the perimeter of the window is 500 inches. Find the values of both h and r when the area is a maximum. Find that maximum area.

πr

r

8,

h

h

$2r$

don't round!
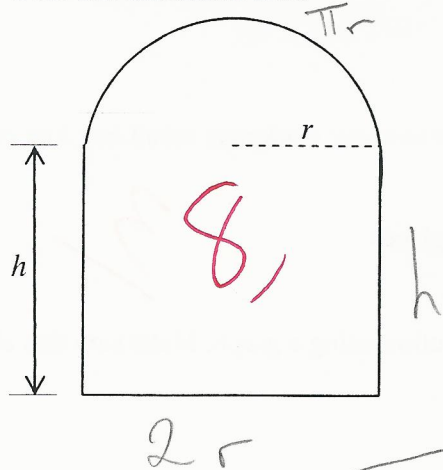
$2h + \boxed{5.14} r = 500$

$2h = 500 - 5.14r$

$\boxed{h} = 250 - 2.57r$

$A = 2rh + \dfrac{\pi r^2}{2} + 2$

$= 2r(250 - 2.57r) + \dfrac{\pi r^2}{2}$

$= 500r - 5.14r^2 + 1.57r^2$

$= -3.57r^2 + 500r$

$h = 250 - 2.57(70.028)$

$= 250 - 179.972$

$= 70.028$

$\text{Vertex} = \dfrac{-b}{2a}, \ f\left(\dfrac{-b}{2a}\right)$

$\dfrac{-500}{-7.14} = 70.028$

$A = -3.57(70.028)^2 + 500(70.028)$
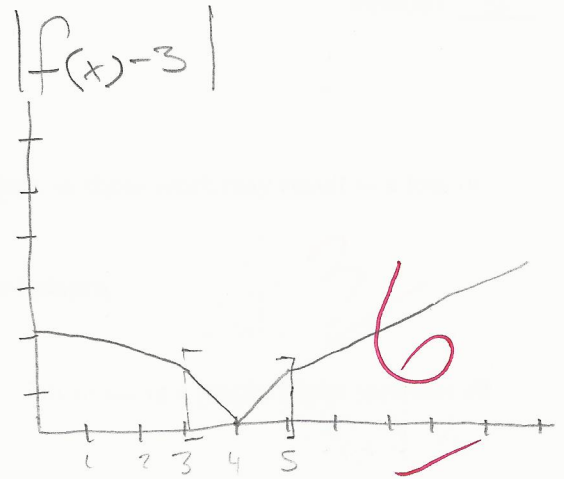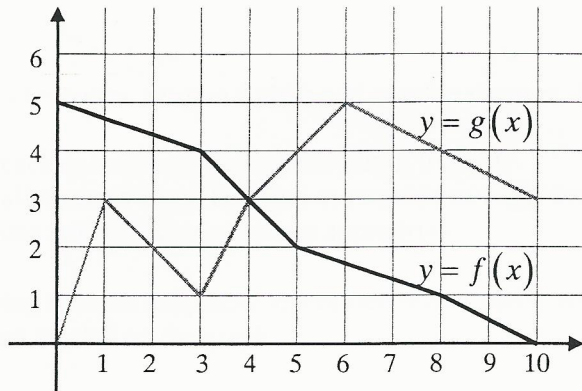
$= -3.57(4903.921) + 35014$

$= -17506.998 + 35014$

$= 17507.003$

When $r = 70.028$ in., the maximum area is achieved.

$\boxed{r = 70.028 \text{ in.} \quad \text{Area} = 17507.003 \text{ in}^2 \\ h = 70.028 \text{ in.}}$

3. For questions 1-9, refer to the graphs of the functions f and g in the following figure. Assume that the domain of each function is $[0,10]$.

$|f(x)-3|$

6

1. For which x-value is $f(x) = g(x)$?   When x=4

2. For which x-value is it true that $g(x) \le f(x)$?   When x is [0, 4)

3. Compute each of the following:
   (a) $(f+g)(8)$ f(8)=2   =6
      +g(8)=4
   (b) $(f-g)(8)$ f(8)=2   =-2
      -g(8)=4
   (c) $(fg)(8)$ f(8)=2 · =8
      xg(8)=4
   (d) $(f/g)(8)$
      $f(8)=2$ $=\frac{1}{2}$
      $g(8)=4$

4. Which is larger, $(f \circ f)(10)$ or $(g \circ g)(10)$?
   f(f(10))=f(5)=0 ; g(g(10))=g(3)=1   1>0 thus: (g∘g)(10) is larger

5. Compute $g[f(10)] - f[g(10)]$.
   g(f(10))=g(0)=5 ; f(g(10))=f(3)=4 ; 5-4= 1

6. For which x-values is it true that $f(x) \ge 3$?
   When x is [0, 4]

7. For which x-values is it true that $|f(x) - 3| \le 1$?
   When x is [3, 5]

8. Specify the coordinates of the highest point on the graph of each of the following equations:
   (a) $y = g(-x)$ (-6,5)
   (b) $y = -g(x)$ (0,0)
   (c) $y = g(x-1)$ (7,5)
   (d) $y = f(-x)$ (0,5)
   (e) $y = -f(x)$ (10,0)
   (f) $y = -f(-x)$ (-10,0)

9. For which values of x in the interval $(4,7)$ is the quantity $\dfrac{f(x) - f(5)}{x-5}$ negative?
   →(4,7) $\dfrac{f(x)-2}{x-5}$

   [5,7) − (4,5)+
   (5,7) + (4,5)−

   X will be negative in the interval (4,5)∪(5,7)