Honors Precalculus
Chapter 3 Exam
Cozean/Fox

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Period: 5

Free Response

Directions: Please show all relevant work in a clear and concise manner. Failure to show work may result in a loss of credit. Also …

1. each question will be graded out of 9 points
2. all answers are to be exact or rounded to three decimal places
3. use interval notation where appropriate

(6)

1. Solve and verify graphically: $\log_3 \frac{x+2}{3x-1} > 0$ $\Rightarrow \frac{x+2}{3x-1} > 1 \Rightarrow \frac{x+2}{3x-1} - 1 > 0$ ↓ solve

$\log_3(x+2) - \log_3(3x-1) > 0$

$\log_3(x+2) - \log_3(x-\frac{1}{3}) > 0$

Critical points: $x = -2, \frac{1}{3}$

+1

$\log_3(x+2) > \log_3(3x-1)$

$x+2 > 3x-1$

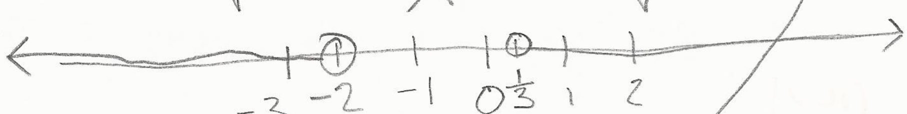
$-2x-3>0$

$x-\frac{3}{2}<0$

$x<\frac{3}{2}$ $\quad \frac{1}{3}<x<\frac{3}{2}$

	-2		$\frac{1}{3}$	
$x+2$	−	+		+
$x-\frac{1}{3}$	−	−		+

✓ ✗ ✓

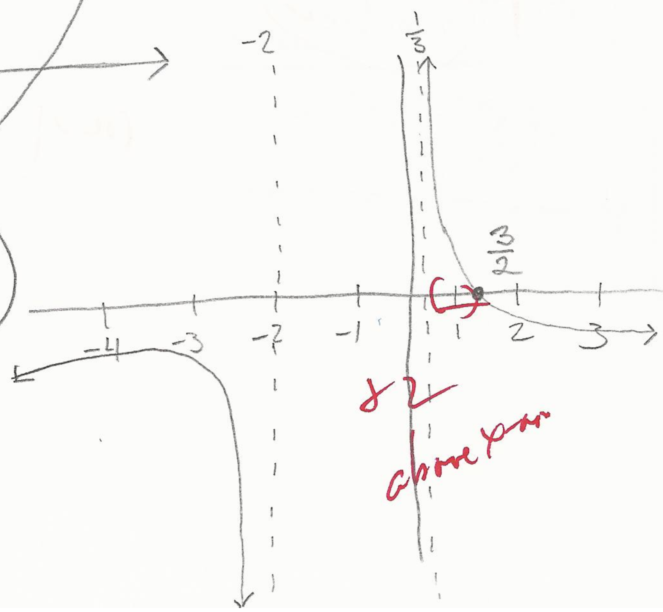


$\left(\frac{1}{3}, \frac{3}{2}\right)$

+2

✓ D: $(-\infty,-2)\cup(\frac{1}{3},\infty)$

+1



+2 above zero

2. Newton's Law of Cooling

Newton's Law of Cooling states that the rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium.

The Law involves what's called a "differential equation" that is studied in Calculus. A form more agreeable to us in our Precalculus class is:

$$T = \frac{1}{k}\ln\left(\frac{I-A}{F-A}\right)$$

where T = cooling time Seconds

k = cooling constant
I = initial temperature F°
F = final temperature F°
A = ambient temperature F°

15 minutes = 900 seconds

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If it takes 15 minutes for a 180° object to cool to 140°, in a 74° room, how much longer will it take for the object to cool to 110°?

$$900 = \frac{1}{K}\ln\left(\frac{180-74}{140-74}\right) \rightarrow 900 = \frac{1}{K}\ln\left(\frac{106}{66}\right) \rightarrow 900 = \frac{0.474}{K} \rightarrow K = \frac{0.474}{900}$$

$K = 0.000\overline{526}$

≈ 0.000527

$$T = \frac{1}{0.000527}\ln\left(\frac{180-74}{110-74}\right) \rightarrow T = \frac{1}{0.000527}\ln\left(\frac{106}{36}\right) \rightarrow T = \frac{1.0799}{0.000527} \rightarrow T = 2049.146$$

It would take 34 minutes and 9.146 seconds to cool this 180° object to 110° in a room with the temperature 74°. That is 19 minutes and 9.146 seconds longer than just cooling it to 140°.

how? -15 minutes

-1

FR -1

3. The half-life of caffeine varies widely among individuals depending on such factors as age, liver function, concurrent medications, etc., with an average time of five to six hours. For purposes of this exam question we'll use a half-life of 5.5 hours. The amount of caffeine in a Rockstar Citrus Punched (16 fl. oz.) is 240 mg. [http://www.cspinet.org/new/cafchart.htm]

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(a) You just chugged a Rockstar Citrus Punched … what is the equation that relates the amount of caffeine in your blood, C, to time, t?

$C(t)=240\cdot\left(\frac{1}{2}\right)^{\frac{t}{5.5}}$ 3/3 t in terms of hours

(b) How much of the initial 240 mg will be present after 8 hours?

$240\cdot\left(\frac{1}{2}\right)^{\frac{8}{5.5}}=\frac{240}{2.741}\approx 87.559$ mg will still be present rnd 1/2

(c) How many hours would it take to reduce the amount of caffeine in your blood to 20 mg (provided you don't ingest more caffeine)?

$20=240\cdot\left(\frac{1}{2}\right)^{\frac{t}{5.5}}$

$\frac{20}{240}=\left(\frac{1}{2}\right)^{\frac{t}{5.5}} \rightarrow \frac{1}{12}=\left(\frac{1}{2}\right)^{\frac{t}{5.5}}$

$\log_{\frac{1}{2}}\frac{1}{12}=\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^{\frac{t}{5.5}} \rightarrow \log_{\frac{1}{2}}\frac{1}{12}=\frac{t}{5.5}$

$t=5.5\log_{\frac{1}{2}}\frac{1}{12}=5.5(3.585)=19.718$ rnd 3/4

It would take 19 hours, 43 minutes, and 3 seconds for the amount of caffeine in your body to reduce to 20 mg