Honors Precalculus
Chapters 4, 5 and 6.1-6.2 Exam
Fox

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Form C

Free Response

Directions: Please show all relevant work in a clear and concise manner. Failure to show work may result in a loss of credit. Also …

1. each question will be graded out of 9 points
2. all answers are to be exact or rounded to three decimal places
3. use interval notation where appropriate

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1. Algebraically find all solutions on the interval $[0, 2\pi)$. Verify graphically by sketching the appropriate graph(s) in the accompanying "screen" and properly indicating/showing the solutions.

$$25\cos^2 x = 13 - 20\sin x$$

I

$25(1-\sin^2 x) = 13 - 20\sin x$

$25 - 25\sin^2 x = 13 - 20\sin x$

$-25\sin^2 x = -12 - 20\sin x$

$-25\sin^2 x + 20\sin x + 12 = 0$

II

$\dfrac{-20 \pm \sqrt{400 - 4(-25)(12)}}{-50}$

$\dfrac{-20 \pm \sqrt{400 + 1200}}{-50}$

$\dfrac{-20 \pm \sqrt{1600}}{-50}$

$\dfrac{-20 \pm 40}{-50}$

$\dfrac{-60}{-50}$ $\dfrac{20}{-50}$

$\cancel{\dfrac{6}{5}}$ $-\dfrac{2}{5}$

$\sin x = -\dfrac{2}{5}$, $\dfrac{6}{5}$

III $\sin^{-1}\left(-\dfrac{2}{5}\right) = \cancel{-0.412}$ out of bounds

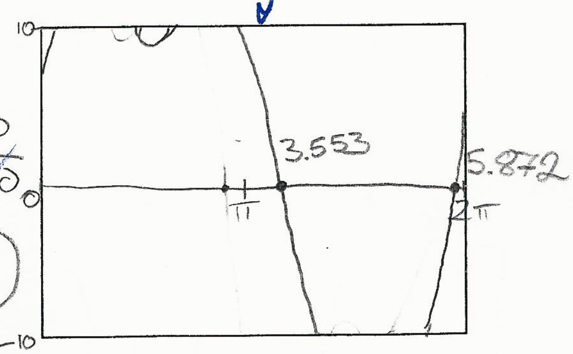
$\pi - (-0.412) = 3.553$

$\sin^{-1}\left(\dfrac{6}{5}\right) = \varnothing$

$2\pi - 0.412 = 5.872$

$$\boxed{x = 3.553, 5.872}$$

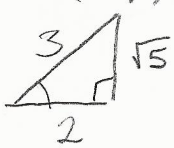
what did you graph?



2. Find the EXACT value and algebraically simplify

$$\sin\left[\cos^{-1}\left(\frac{2}{3}\right)+2\sin^{-1}\left(-\frac{1}{3}\right)\right]$$

$$\sin\left(\cos^{-1}\left(\frac{2}{3}\right)\right)\cos\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)+\cos\left(\cos^{-1}\left(\frac{2}{3}\right)\right)\sin\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)$$



$$\frac{\sqrt{5}}{3}\cdot\cos\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)+\left(\frac{2}{3}\right)\cdot\sin\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)$$

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$$\frac{\sqrt{5}}{3}\cdot\left(1-2\sin^2\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)\right)+\left(\frac{2}{3}\right)\cdot 2\sin\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)\cos\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)$$

$$\frac{\sqrt{5}}{3}\cdot\left(1-2\left(-\frac{1}{3}\right)^2\right)+\left(\frac{2}{3}\right)\cdot 2\left(-\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

$$\frac{\sqrt{5}}{3}\cdot\left(1-\frac{2}{9}\right)+\left(\frac{2}{3}\right)\cdot\left(\frac{-4\sqrt{2}}{9}\right)$$

$$\left[\frac{\sqrt{5}}{3}\cdot\frac{7}{9}\right]+\left[\frac{2}{3}\cdot\frac{-4\sqrt{2}}{9}\right]$$

$$\frac{7\sqrt{5}}{27}-\frac{8\sqrt{2}}{27}$$

$$\boxed{\frac{7\sqrt{5}-8\sqrt{2}}{27}}=\sin\left[\cos^{-1}\left(\frac{2}{3}\right)+2\sin^{-1}\left(-\frac{1}{3}\right)\right]$$

3. Algebraically find all solutions on the interval $[0, 2\pi)$. Verify graphically by sketching the appropriate graph(s) in the accompanying "screen" and properly indicating/showing the solutions.

$$\cos 2x \cos x - \sin 2x \sin x = \frac{\sqrt{3}}{2}$$

$[0, 2\pi)$

$\cos(2x + x)$

$\cos(3x) = \frac{\sqrt{3}}{2}$

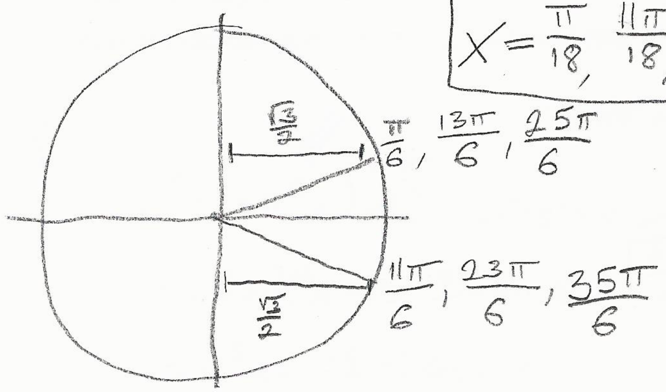
All valid within $[0, 2\pi)$

$\cos(x) = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6} \ldots$

$\div 3$

$$\boxed{x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}}$$



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