3:17

$\frac{47}{50}$ " $\overline{27}$

Honors Precalculus Trigonometry Free Response
Fox

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09 January 2009

2013-12-11

A calculator IS allowed (and is necessary).

<u>*Show all relevant work in a clear and concise manner*</u>. <u>*Failure to show work will result in a loss of credit*</u>.

✓ Prove the following identity:

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$$

① $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

② $\frac{1+\sin\theta}{\cos\theta} \cdot \frac{1-\sin\theta}{1-\sin\theta}$

③ $\frac{1-\sin^2\theta}{\cos\theta - \cos\theta\sin\theta}$

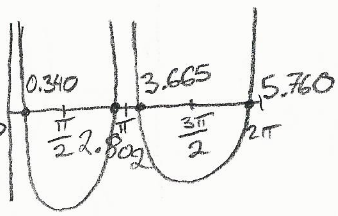
④ $\frac{\cos^2\theta}{\cos\theta - \cos\theta\sin\theta}$

⑤ $\cancel{\frac{\cos\theta}{\cos\theta}} \cdot \boxed{\frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}}$

10/10

Solve *each* of the following equations on the interval $[0, 2\pi)$, and then verify graphically. Give exact answers, if possible, or to three decimal places.

$$\cot^2 x = 5 + \csc x$$



$\cot^2 x - \csc x - 5 = 0$

$(\csc^2 x - 1) - \csc x - 5 = 0$

$\csc^2 x - \csc x - 6 = 0$

$(\csc x - 3)(\csc x + 2) = 0$ wrong

$\csc x = -2$ 3 unit circle

$\sin x = -1/2$ $\sin x = 1/3$

$x = \csc^{-1}(-2), \csc^{-1}(3)\ [0, 2\pi)$

$\csc^{-1}(-2) = -0.524$

$\csc^{-1}(3) = 0.340$

$\pi - \csc^{-1}(-2) = \underline{3.665}$

$2\pi + \csc^{-1}(-2) = \underline{5.760}$

$\csc^{-1}(3) = \underline{0.340}$

$\pi - \csc^{-1}(3) = \underline{2.802}$

$\boxed{x = 3.665, 5.760, 0.340, 2.802}$ $= 7\pi/6$, $= 11\pi/6$ see -1

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$$4\sin^2 2t - 1 = 0$$

$(2\sin 2t + 1)(2\sin 2t - 1) = 0$

$\sin 2t = \frac{1}{2}, -\frac{1}{2}$

$\sin^{-1}\left(\frac{1}{2}\right) =$

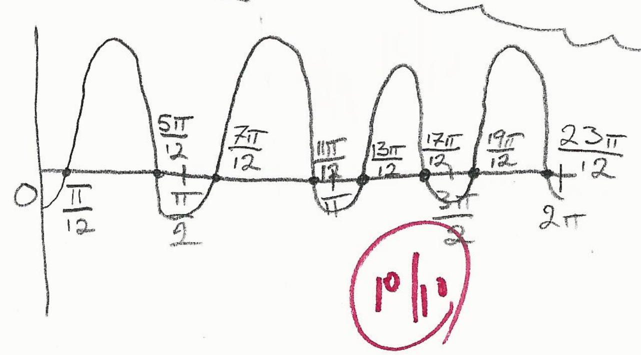
$2t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$\sin^{-1}\left(-\frac{1}{2}\right) =$

$2t = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$\div 2$

$\boxed{t = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}}$



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47/50

The end of a plastic ruler is vibrating in damped harmonic motion. A function that relates the position (distance above or below the center of vibration in cm) of the end of the ruler to time (in sec) is given by

$$d(t) = -1.2e^{-9t}\cos(100\pi t)$$

(a) Where is the end of the ruler when $t = 0$?
(b) During the first 0.1 seconds, what is the ruler's maximum distance, to three decimal places, *above* the centre of vibration?
(c) At what time does the ruler *first* go through the central position?

a) -1.2 cm   1.2 cm below the centre of vibration

b) 1.1 cm   1.097 3dp

c) 0.005 seconds

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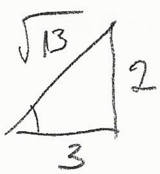
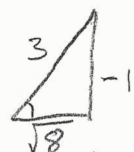
Find the <u>exact</u> value of the following:



$$\cos\left[\tan^{-1}\left(\frac{2}{3}\right) + 2\sin^{-1}\left(-\frac{1}{3}\right)\right]$$



$$\cos\left(\tan^{-1}\left(\frac{2}{3}\right)\right)\cos\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right) - \sin\left(\tan^{-1}\left(\frac{2}{3}\right)\right)\sin\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)$$

$$\frac{3}{\sqrt{13}}\cos\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right) - \frac{2}{\sqrt{13}}\sin\left(2\sin^{-1}\left(-\frac{1}{3}\right)\right)$$

$$\frac{3}{\sqrt{13}}\left(1-2\sin^2\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)\right) - \frac{2}{\sqrt{13}}\left(2\sin\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)\cos\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)\right)$$

$$\frac{3}{\sqrt{13}}\left(1-2\left(-\frac{1}{3}\right)^2\right) - \frac{2}{\sqrt{13}}\left(2\cdot\left(-\frac{1}{3}\right)\cdot\frac{\sqrt{8}}{3}\right)$$

$$\frac{3}{\sqrt{13}}\cdot\frac{7}{9} - \frac{2}{\sqrt{13}}\cdot\frac{-2\sqrt{8}}{9}$$

$$\frac{21}{9\sqrt{13}} + \frac{4\sqrt{8}}{9\sqrt{13}}$$

$$\boxed{\frac{21+4\sqrt{8}}{9\sqrt{13}}}$$

10/10

The end of a plastic ruler is vibrating in damped harmonic motion. A function that relates the position (distance above or below the centre of vibration in cm) of the end of the ruler to time (in sec) is given by

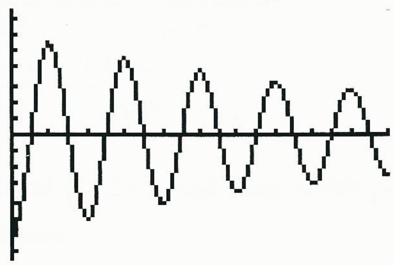
$$d(t) = -1.2e^{-9t}\cos(100\pi t)$$

(a) Where is the end of the ruler when $t = 0$?
(b) During the first 0.1 seconds, what is the ruler's maximum distance, to three decimal places, *above* the centre of vibration?
(c) At what time does the ruler *first* go through the central position?

(a) $d(0) = -1.2e^{-9(0)}\cos(100\pi(0)) = -1.2$ so the end of the ruler is 1.2 cm *below* the center of vibration.

For parts b and c, graph the function and use the calculator's "CALC" capabilities.

Xmin=0
Xmax=0.1
Xscl=0.01
Ymin=-1.5
Ymax=1.5
Yscl=0.2



(b) use CALC, maximum to find *first* relative maximum . . . 1.097 cm

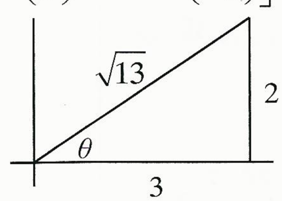
(c) use CALC, zero to find first t-intercept . . . 0.005 sec OR $d(t) = -1.2e^{-9t}\cos(100\pi t) = 0$

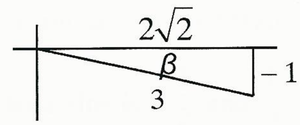
$$\Rightarrow \cos(100\pi t) = 0 \Rightarrow 100\pi t = \frac{\pi}{2} \Rightarrow t = \frac{1}{200}\text{ sec}$$

Find the <u>*exact*</u> value (do not simplify) of the following:

$$\cos\left[\tan^{-1}\left(\frac{2}{3}\right) + 2\sin^{-1}\left(-\frac{1}{3}\right)\right]$$

Let $\theta = \tan^{-1}\left(\frac{2}{3}\right)$ and $\beta = \sin^{-1}\left(-\frac{1}{3}\right)$.





Substituting . . .

$$\cos\left[\tan^{-1}\left(\frac{2}{3}\right) + 2\sin^{-1}\left(-\frac{1}{3}\right)\right] = \cos(\theta + 2\beta)$$

$$\cos(\theta + 2\beta) = \cos\theta\cos 2\beta - \sin\theta\sin 2\beta$$
$$= \cos\theta\left(1 - 2\sin^2\beta\right) - \sin\theta\left(2\sin\beta\cos\beta\right)$$
$$= \left(\frac{3}{\sqrt{13}}\right)\left[1 - 2\left(-\frac{1}{3}\right)^2\right] - \left(\frac{2}{\sqrt{13}}\right)(2)\left(-\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$$

Therefore, $\cos\left[\tan^{-1}\left(\frac{2}{3}\right) + 2\sin^{-1}\left(-\frac{1}{3}\right)\right] = \left(\frac{3}{\sqrt{13}}\right)\left[1 - 2\left(-\frac{1}{3}\right)^2\right] - \left(\frac{2}{\sqrt{13}}\right)(2)\left(-\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$

$$= \frac{21 + 8\sqrt{2}}{9\sqrt{13}} \approx 0.996$$

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Fox

09 January 2009

Prove the following identity:

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$$

$$\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{1-\sin\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$\frac{1+\sin\theta}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}$$

Solve *each* of the following equation on the interval $[0, 2\pi)$, and then verify graphically. Give exact answers, if possible, or to three decimal places.

$\cot^2 x = 5 + \csc x$

$\csc^2 x - 1 = 5 + \csc x$

$\csc^2 x - \csc x - 6 = 0$

$(\csc x - 3)(\csc x + 2) = 0$

$\sin x = -\frac{1}{2}$ or $\sin x = \frac{1}{3}$

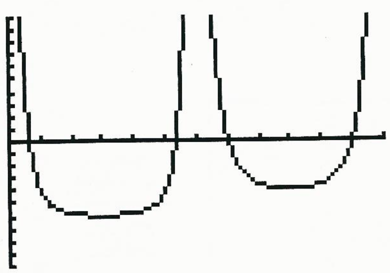
$\sin x = -\frac{1}{2}$ produces $x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

Using a calculator, $x = \sin^{-1}\left(\frac{1}{3}\right) \approx 0.340$

This value IS between $[0, 2\pi)$ and the other angle whose sine is 0.340 is $\pi - 0.340 \approx 2.801$

So the solution set is $\left\{0.340, 2.801, \frac{7\pi}{6}, \frac{11\pi}{6}\right\}$

Graph $y = \cot^2 x - 5 - \csc x$ and find x-intercepts.



$4\sin^2 2t - 1 = 0$

$\sin^2 2t = \frac{1}{4}$

$\sin 2t = \pm\frac{1}{2}$

$0 \le t < 2\pi \Rightarrow 0 \le 2t < 4\pi$

So $2t$ equals all the angles, between 0 and 4π whose sine is $-\frac{1}{2}$ and $\frac{1}{2}$

$$2t = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$$

$$t = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

Graph $y = 4\sin^2 2t - 1$ and find x-intercepts.