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PreCalculus Honors

Chapter 9 and 10 Summary

- Sequence: a set of numbers that follow a mathematic pattern
- Series: a set of numbers that are added together, following a mathematic pattern
- Sequence: $A_n = A(n)$; Series: $S_n = \sum_{i=1}^{n} s(i)$
- Recursive sequence: given "seed terms", further terms are derived from previous terms
- $n! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot \ldots \cdot (n-1) \cdot n$
- Arithmetic (addition)
  - Sequence
    - nth term: $a_n = dn + c$ or $a_n = a_1 + (n-1)d$
    - Sum of a finite sequence: $\frac{n}{2}(a_1 + a_n) = S_n$
    - Common difference: $d = a_n - a_{(n-1)}$
- Geometric (multiplication)
  - Sequence/series
    - nth term: $a_n = a_1 r^{n-1}$
    - sum of a finite sequence: $S_n = \sum_{i=1}^{n} a_1 r^{i-1} = a_1\left(\frac{1-r^n}{1-r}\right)$
    - Sum of an infinite series: $S_n = \sum_{i=1}^{\infty} a_1 r^i = \frac{a_1}{1-r}$
    - Common ratio: $\frac{a_n}{a_{n-1}} = r$ — if $r \leq -1$ or $r \geq 1$, divergent series; if $-1 > r > 1$, convergent series
- Binomial Theorem

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

n^{th} term: $\binom{n}{r-1} a^{n-(r-1)} b^{(r-1)}$

$\binom{n}{r} = {}_nC_r =$ "n choose r" $= \frac{n!}{r!(n-r)!}$

"Binomial Coefficient"

Pascal's Triangle

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \ldots \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

- $(n+1)$ terms
- Starts with a^n, ends with b^n
- $b^{\square}$ increases →, $a^{\square}$ decreases →
- for any term $a^{\boxed{\circ}} + b^{\boxed{\circ}} = n$

Conics!: the intersection of a plane and a double-napped cone

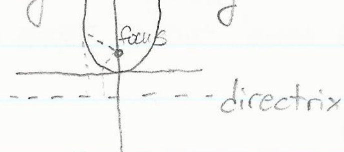
- defined as a locus of points
- general form: $Ax^2 + By^2 + Cx + Dy + E = 0$

- Complete the square to move into Standard form

Parabola:

$(x-h)^2 = 4p(y-k)$

focus: p



Circle:

$(x-h)^2 + (y-k)^2 = r^2$

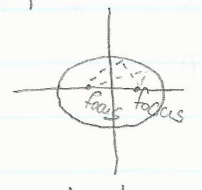
focus: 0

$* = 0$: (degenerative) point

$= -x$: (degenerative) empty set $\varnothing$

Ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$



$* = 0$: (degenerative) point

$= -x$: (degenerative) empty set $\varnothing$

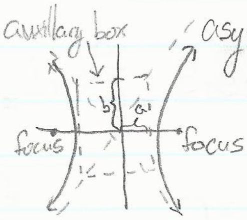
major axis: x when $a>b$, y when $b>a$

eccentricity: $e = \frac{c}{a}$

focus: $c = \sqrt{a^2-b^2}$

Hyperbola:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



asymptotes: $y = k \pm \frac{b}{a}(x-h)$

$* = 0$: (degenerative) intersecting lines

transverse axis: x when positive, y when positive

eccentricity: $e = \frac{c}{a}$

focus: $c = \sqrt{a^2+b^2}$

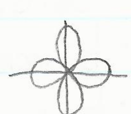
- Parametric Equations: $x = f(t)$, $y = g(t)$; functions of t
  - eliminate the parameter by solving for t, then plugging it in to the other equation
- Polar Coordinates
  - (r, θ)
  - $r\cos(\theta) = x$
  - $r\sin(\theta) = y$
  - $r^2 = x^2 + y^2$
  - $\tan\theta = \frac{y}{x}$

Symmetry tests: replace (r,θ) with $(-r,-\theta)$ = about the $\theta = \frac{\pi}{2}$ axis

replace (r,θ) with $(r,-\theta)$ = about the polar axis

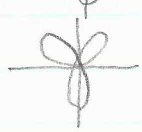
replace (r,θ) with $(-r,\theta)$ = about the pole

- Rose Curves: $r = \cos(2\theta)$



$(r = a\cos(n\theta))$

$r = \sin(3\theta)$



if n even: $2n$ petals

if n odd: n petals

- Circles: $r = a\cos(\theta)$

$r = a\sin(\theta)$

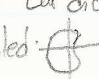
- Limaçon

$r = a + b\cos(\theta)$

intercepts: $b \pm a$

$\frac{a}{b} < 1$: with loop 

$\frac{a}{b} = 1$: Cardiod 

$1 < \frac{a}{b} < 2$: dimpled  ; $\frac{a}{b} > 2$: Convex